Math 2362: Advanced Engineering Mathematics

Credit hours: **4 Credit Hours**
Prerequisites: **Math 2243 with a grade of C or better**

Course Description

This course covers first-order ordinary differential equations and second-order linear differential equations. Methods for solving differential equations are studied, including the use of Laplace transforms and power series solutions. In addition to differential equations, students are introduced to matrices and linear algebra, as well as functions of a complex variable. This course transfers to URI as either Math 244 or Math 362.

Course Objectives

1. Introduce students to ordinary differential equations and the methods for solving these equations
2. Use differential equations as models for real world phenomena
3. Integrate the knowledge accumulated in the calculus sequence to solve applied problems
4. Introduce the fundamentals of Linear Algebra and Complex Analysis
5. Provide a rigorous introduction to upper level mathematics which is necessary for students of engineering, physical sciences and mathematics

Learning Outcomes

1. Utilize various methods for solving ODEs
2. Solve initial value problems and understand the existence and uniqueness of such solutions
3. Recognize ODEs of varying order and use these to solve problems involving population dynamics, oscillation of a spring and resistance in a circuit
4. Work with and solve homogeneous and non-homogeneous ODEs and systems of ODEs
5. Learn additional methods for solving ODEs including Euler's method, the power series method and Laplace transforms
6. Perform basic operations with matrices
7. Find the inverse of a matrix, determinant of a square matrix, as well as eigenvalues and eigenvectors and investigate associated applications
8. Use matrices to solve systems of equations
9. Express complex numbers in trigonometric and polar form
10. Perform operations with complex numbers, including finding the roots of unity
11. Explore functions of a single complex variable
12. Calculate derivatives of analytic functions
13. Calculate line integrals in the complex plane
14. Study Cauchy-Riemann equations, Cauchy's integral theorem and Cauchy's integral formula

Course Topics

I. FIRST ORDER ORDINARY DIFFERENTIAL EQUATIONS (ODES)

- A. Basic concepts and modeling
- B. Slope fields and Euler's method
- C. Separation of variables
- D. Integrating factors
- E. Linear ODEs and applications to population dynamics
- F. Existence and uniqueness of solutions for initial value problems

II. SECOND ORDER LINEAR ODES

A. Second order homogeneous linear ODEs with constant coefficients
B. Modeling of free oscillations of a mass-spring system
C. Euler-Cauchy equations
D. Existence and uniqueness of solutions
E. Nonhomogeneous ODEs
F. Modeling: forced oscillations and electric circuits
G. Solutions by variation of parameters

III. HIGHER ORDER LINEAR ODES

A. Homogeneous linear ODEs
B. Nonhomogeneous linear ODEs

IV. SYSTEMS OF ODES

A. Basics of matrices and vectors
B. Basic theory of systems of ODEs and the Wronskian
C. Constant-coefficient systems
D. Criteria for critical points and stability
E. Nonhomogeneous linear systems of ODEs

V. OTHER METHODS OF SOLVING ODES

A. The power series method
B. The Laplace transform method

VI. INTRODUCTION TO LINEAR ALGEBRA

A. Operations with matrices and vectors
B. Matrix multiplication
C. Linear systems of equations and Gauss elimination
D. Linear independence and the rank of a matrix
E. Determinants and Cramer's rule
F. Inverse of a matrix
G. Vector spaces
H. Finding eigenvalues and eigenvectors

VII. FUNCTIONS OF A COMPLEX VARIABLE

A. Complex numbers in the plane
B. Trigonometric and polar forms of a complex number
C. Powers and roots of unity
D. Analytic functions and the derivative
E. Cauchy-Riemann equations
F. Exponential, trigonometric and hyperbolic functions
G. Euler's formula
H. Logarithms
I. Line integrals in the complex plane
J. Cauchy's integral theorem and integral formula
K. Derivatives of analytic functions