Math 2243: Calculus III

Credit hours: **4 Credit Hours**
Prerequisites: **Math 2142 with a grade of C or better**

Course Description

This course covers the calculus of three-dimensional space, including partial derivatives, multiple integrals and the calculus of vector-valued functions.

Course Objectives

1. Apply the differential and integral calculus learned in earlier courses to problems involving several variables
2. Introduce students to the three dimensional Euclidian space and multi-dimensional vectors
3. Use partial derivatives and multiple integrals to solve problems in pure mathematics and various applications
4. Prepare students for differential equations, linear algebra, complex analysis, and other advanced topics
5. Prepare Engineering and Science students for the mathematics required in their upper level course work

Learning Outcomes

1. Plot points, calculate distances and mid-points, and work with vectors in the three dimensional space
2. Utilize vectors as a mathematical tool for solving both pure and applied problems
3. Explore the properties of vector functions and use these functions to model problems involving motion
4. Find the domain and range of scalar functions of several variables and sketch three dimensional surface graphs and two dimensional contour plots of simple scalar functions
5. Compute limits and discuss the continuity of scalar functions of several variables
6. Calculate partial derivatives of the first and second order
7. Calculate the gradients of scalar functions and use them to solve problems requiring a directional derivative
8. Solve problems involving rates of change and solve optimization problems
9. Use double and triple integrals to calculate volumes and solve application problems
10. Identify regions of integration within the domain and re-write the order of integration for multiple integrals
11. Perform a change of variables utilizing polar, cylindrical or spherical coordinates to calculate integrals
12. Explore vector fields and complete fundamental calculations
13. Determine if a vector field is conservative and find the potential scalar function of a conservative vector field
14. Calculate line integrals and apply the information contained in Green's Theorem and Stokes' Theorem

Course Topics

I. GEOMETRY OF THE EUCLIDIAN SPACE

- A. Plot points in the three dimensional space
- B. Calculate distances and mid-points in the three dimensional space
- C. Write the equation of a sphere in space given a center point and radius
- D. Sketch the surface generated by solution set of an equation of three variables
- E. Work with points and equations given in polar coordinates and convert between rectangular and polar coordinates
- F. Work with points and equations given in cylindrical or spherical coordinates
- G. Convert between rectangular coordinates and cylindrical or spherical coordinates

II. VECTORS IN THE TWO AND THREE DIMENSIONAL SPACES

- A. Visualize vectors in two or three dimensions

B. Find the sum or difference of vectors
C. Find the dot product of two vectors and use it to find the angle between vectors
D. Find the projection of one vector onto another and use this to solve inclined plane problems
E. Calculate cross products and determinants and use the results to solve area, volume and torque problems
F. Use vectors as a tool to solve other mathematical problems
G. Use vectors and knowledge of the geometry of the Euclidian space to work with lines and planes in the three dimensional space

III. VECTOR FUNCTIONS

A. Parameterize equations of two variables and express as a vector function
B. Sketch paths in the plane and space generated by vector functions
C. Find the domain of a vector function and calculate limits, derivatives and antiderivatives of vector functions
D. Use vector functions as mathematical models for the motion of an object through a plane or space
E. Calculate the velocity, acceleration and speed of an object in motion
F. Use integration to solve initial value problems

IV. SCALAR FUNCTIONS OF SEVERAL VARIABLES

A. Find the domain of a function of several variables and sketch the domain of a function of two independent variables
B. Sketch surfaces generated by simple functions of two independent variables
C. Sketch contour plots of simple functions of two independent variables
D. Calculate limits of functions of several variables
E. Discuss the continuity of a function of several variables

V. DIFFERENTIATION

A. Calculate the first and second order partial derivatives of a function of several variables
B. Calculate the gradient of a function of several variables
C. Compute the directional derivative of a function of several variables and use it to solve problems involving rates of change
D. Find the critical points of a function of two variables and classify the points as maximums or minimums
E. Solve basic max/min problems
F. Use Lagrange multipliers to solve constrained optimization problems*
G. Find the equation of a plane tangent to a surface and find the parametric equations for a line perpendicular to a surface

VI. INTEGRATION

A. Calculate a double integral with numerical and variable limits of integration
B. Sketch the region of integration for a double integral
C. Re-write the order of integration for a double integral problem
D. Perform a change of variables and use polar coordinates to evaluate a double integral
E. Use a double integral to calculate areas and volumes
F. Calculate a triple integral with numerical and variable limits of integration
G. Use a triple integral to calculate the volume of a three dimensional solid
H. Perform a change of variables and use cylindrical or spherical coordinates to evaluate a triple integral
I. Use integrals to solve problems involving center of mass, work, and probability
J. General changes of variables and the Jacobian*

VII. INTRODUCTION TO VECTOR FIELDS

A. Sketch a simple vector field and calculate the divergence and the curl
B. Determine if a vector field is conservative and find the potential function of a conservative vector field
C. Calculate a line integral over a path in two or three dimensions
D. Use Green's Theorem, Stokes' Theorem, The Divergence Theorem and The Fundamental Theorem of Line Integrals to solve problems

*Optional